B.Tech.

Fifth Semester Examination

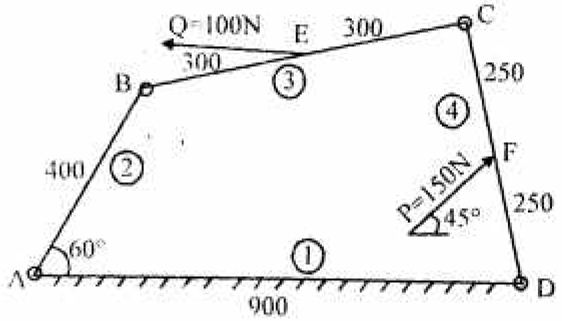
Dynamics of Machines (ME-301-F)

Note : Attempt any five questions.

Q. 1. The dimensions of a four-link mechanism are AB = 400mm, BC = 600mm, CD = 500 mm, AD = 900 mm, and $\angle DAB = 60°$. AD is a fixed link. E is the point on link BC such that BE = 400 mm and CE = 300mm (BEC clockwise).

A force of 150 $\angle 45°$ N acts on DC at a distance of 250 mm from D. Another force of magnitude 100 $\angle 180°$ N acts at point E. Find the required input torque on link AB for static equilibrium of the mechanism.

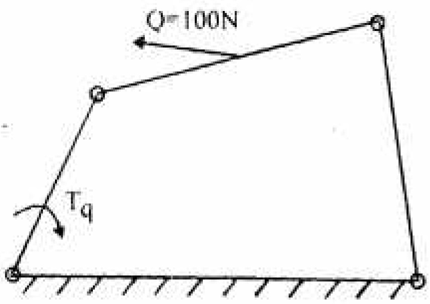
Ans.



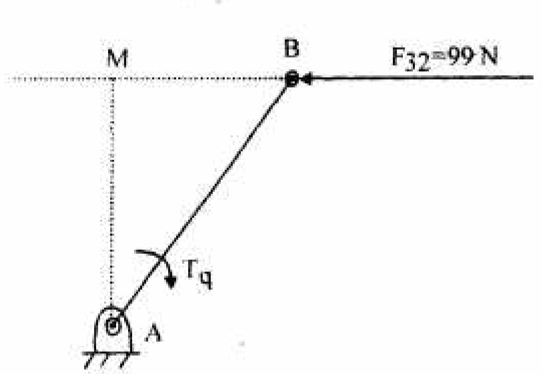
Graphical Method of Force Analysis :

Considering forces applied on links one by one.

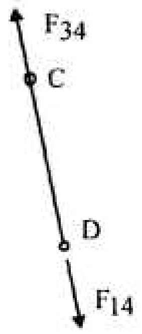
(a) Firstly considering force Q. The configuration diagram will be



F.B.D. for different links



Configuration Diagram of AB

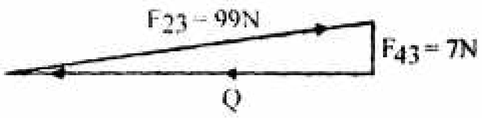


FBD of CD

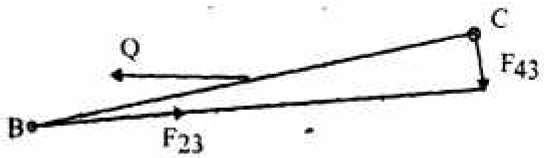
$$T_q = F_{32} \times AM$$

$$T_q = \frac{99 \times 325}{1000}$$

$$= 32.2 \text{ Nm clockwise}$$



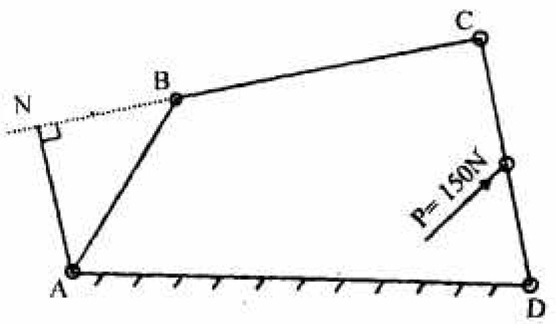
Force diagram for link BC

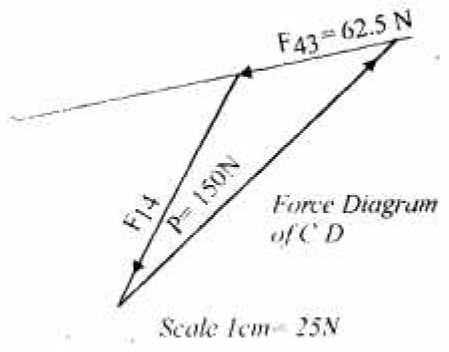


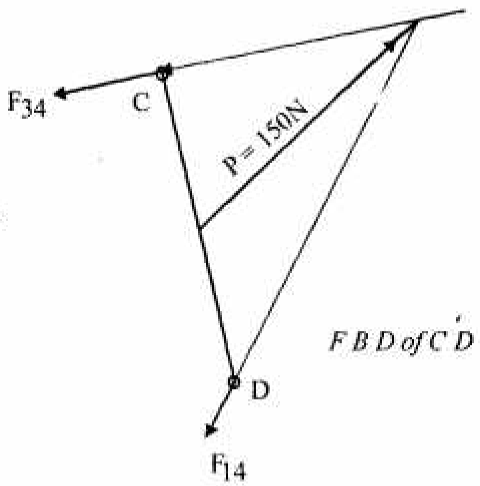
FBD of BC

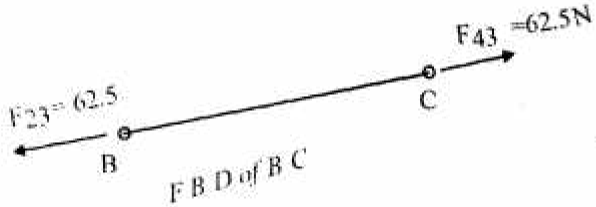
For Force Q

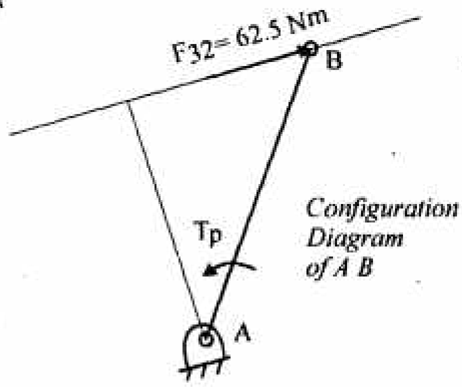
(b) Torque required to balance force 'P'.











$$T_p = F_{32} \times AN \text{ Anticlockwise}$$

$$= 62.5 \times 290 \text{ N} - \text{mm}$$

$$= 18.1 \text{ N m Anticlockwise}$$

(c) Net torque on link AB required.

$$T = T_q + T_p = 32.2 - 18.1 = 14.1 \text{ N} - \text{m clockwise.}$$

Q. 2. The pressure of the front end and back end of piston in an engine is 5.5 and 0.4 Kgf/cm^2 respectively. The diameter of piston and stroke are 30 cm and 40 cm respectively. The engine speed is 300 rpm and weight of reciprocating parts is 50 kg. If connecting rod is 4 times the crank and if piston rod diameter is 2 cm, find the piston effort when the crank has rotated 40° from inner dead centre.

Ans. Given :

$N = 300$ rpm

$\theta = 40°$

$L = 40\text{cm} \Rightarrow r = 20\text{cm}$

$dp = 30$ cm

$$n = \frac{l}{r} = 4$$

$$M_R = 50kg$$

$$P_1 = 5.5 kgf / cm^2$$

$$P_2 = 0.4 kgf / cm^2$$

$$\Delta P = P_1 - P_2$$

$$= 5.1 kgf / cm^2$$

$$\boxed{\Delta P = 50 N / cm^2}$$

$$\omega = \frac{2 \times 3.14 \times 300}{60} = 31.4 rad / s$$

Force on the piston $$F_P = \frac{\pi dp^2 (\Delta P)}{4}$$

$$= \frac{3.14 \times (30)^2 (50N / cm^2)}{4}$$

$$= \frac{3.14 \times (30)^2 \times 50}{4}$$

$$\boxed{F_P = 35300 N}$$

$$F_P = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$F_P = (31.4)^2 \times (0.20) \left[\cos 40 + \frac{\cos 80}{4} \right]$$

$$F_P = (31.4)^2 \times (0.20) \times [0.766 + 0.0434]$$

$$\boxed{F_P = 160 m / s^2}$$

Inertia force due to reciprocating parts,

$$F_i = M_r F_p = 50 \times 160 = 7980 N$$

$$\boxed{F_i = 7980 N}$$

Piston effort, $$F = F_p - F_i$$

$$\boxed{F = 27300 N}$$ **Ans.**

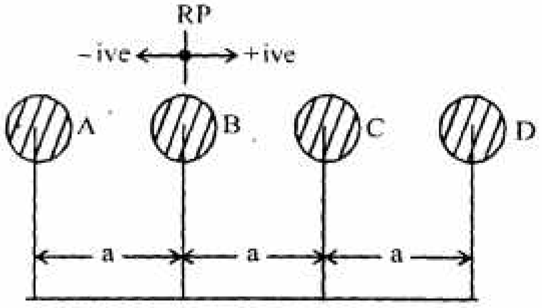
Q. 3. Four masses A, B, C and D revolve at equal radii and are equally spaced along a shaft. The mass B is 7kg and the radii of C and D make angles of 90° and 240° respectively with the radius of B. Find the magnitude of masses A, C and D and the angular position of A so that the system may be completely balanced.

Ans. Reference name.

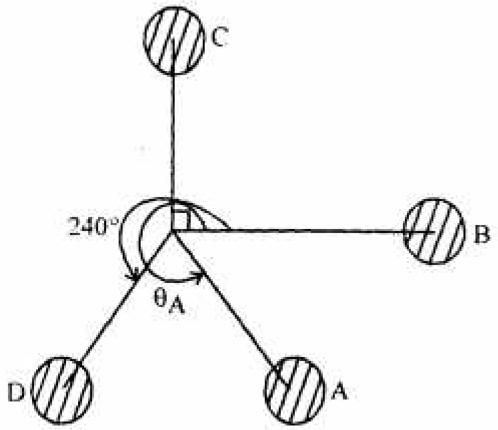
$M_B = 7\text{ kg}$

Let $r = 1\text{ m}$

Plane	M	r meta	M_r m	θ	$M_r \cos\theta$	$M_r \sin\theta$
A	M_A	1	M_A	θ_A	$M_A \cos\theta_A$	$M_A \sin\theta_A$
B	7	1	7	0	7	0
C	M_C	1	M_C	90	0	$M_C \sin 90 (= M_C)$
D	M_D	1	M_D	240	$M_D \cos 240$ $(-0.5 M_D)$	$M_D \sin 240$ $(= -0.866 M_D)$



Position of planes



Position of masses

Solving the problem analytically, we have

$\Sigma M_i r_i \cos\theta_i = 0$ For complete balance

$\Sigma M_i r_i \sin\theta_i = 0$ For complete balance

$\Sigma M_i r_i l_i \cos\theta_i = 0$ For complete balance

$\Sigma M_i r_i l_i \sin\theta = 0$ For complete balance

$\Rightarrow M_A \cos\theta_A - 0.5 M_D + 7 = 0$ ...(i)

$M_A \sin\theta_A + M_C - 0.866 M_D = 0$ ...(ii)

$-M_A a \cos\theta_A - aM_B = 0$ ...(iii)

$-M_A a \sin\theta_A + aM_C - 1.73a M_D = 0$ ...(iv)

Solving the above four equations, we have

$\boxed{M_D = 4.67\,kg}$

$\boxed{M_C = 6.06\,kg}$

$\boxed{M_A = 5.09\,kg}$

$\boxed{M_B = 7\,kg(given)}$

$\theta_A = 157°$ or $203°$

$\Rightarrow \boxed{\theta_A = 203°}$.

Q. 4. (a) What are in-line engines? How are they balanced? Is it possible to balance them completely?

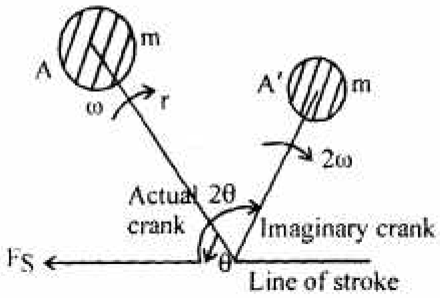
Ans. In-line Engines : The multi cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as in-line engines. The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :

(i) The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must close.

(ii) The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

In order to give the primary balance of reciprocating parts of a multicylinder engine, it is convenient to imagine the reciprocating masses to be transferred to the respective crankpins and to treat the problem as one of the revolving masses.

In order to balance secondary forces of multicylinder in-line engines, the secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at an imaginary crank of length $r/4\pi$ and revolving at twice the speed of the actual crank (i.e., 2ω) as shown.



Secondary Force

The following 2 conditions must be satisfied in order to give a complete secondary balance of an engine :

(i) The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close and.

(ii) The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

For a multicylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided that the number of cranks are not less than four.

The secondary forces cannot be completely balanced for obvious reasons of imbalance of secondary forces in the case of reciprocating masses. The closing side of the secondary force polygon gives the maximum unbalanced secondary force and the closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.

Q. 4. (b) A three cylinder radical engine driven by a common crank has the cylinders spaced at 120°. The stroke is 125 mm, length of the connecting rod 225 mm and the mass of the reciprocating parts per cylinder 2 kg. Calculate the primary and secondary forces at crank shaft speed of 1200 r.p.m.

Ans. Given stroke $= 125\,\text{mm}$

$\Rightarrow$ $$r = \frac{S}{2} = \frac{125}{2} = 62.5\,\text{m}$$

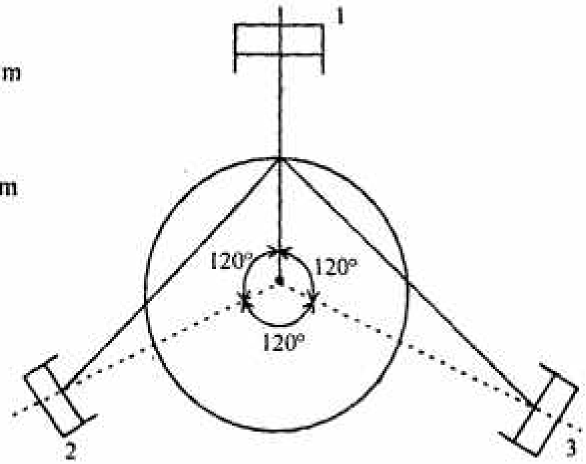
$$= 0.0625\,\text{m}$$

$$l = 225\,\text{mm} = 0.225\,\text{m}$$

$$M = 2\,\text{kg}$$

$$N = 1200\,\text{rpm}$$

$$n = \frac{l}{r} = \frac{225}{62.5} = 3.6$$



We know that the maximum primary force,

$$= 1.5\,Mr\omega^2$$

$$= 1.5 \times 2 \times (0.0625) \times \left(\frac{2\pi \times 1200}{60}\right)^2$$

$$= 2960\,\text{N}$$

$$B_1b_1 = 1.5\,Mr$$
$$= 1.5 \times 2 \times 0.0625$$
$$= 0.188\,N/m$$

Maximum secondary force $= 1.5M \times (2\omega)^2 \times \frac{r}{4n}$

$$= 1.5 \times 2 \times \left(4\pi \times \frac{1200}{60}\right)^2 \times \frac{0.0625}{4 \times 3.6}$$

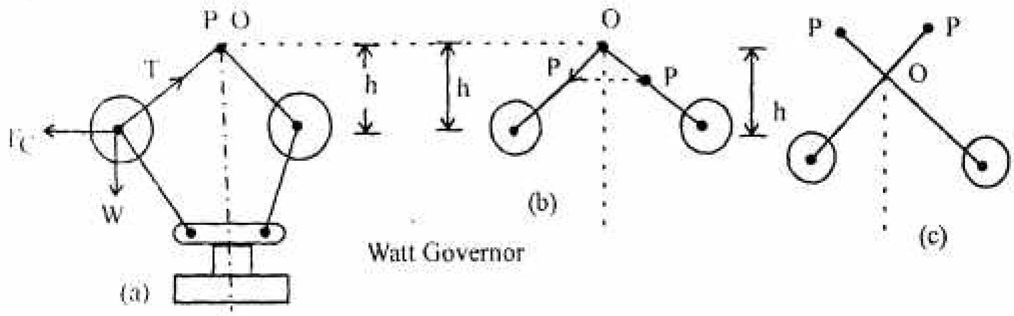
Maximum secondary force $= 822\ N$

$$B_2b_2 = \frac{1.5Mr}{4n} = \frac{1.5 \times 2 \times 0.0625}{4 \times 3.6} = 1.3 \times 10^{-2}\,N-m$$

$\Rightarrow$ $B_2b_2 = 0.013\,N-m$

Q. 5. (a) Explain the term height of the governor. Derive an expression for the height in the case of a watt governor. What are the limitations of a watt governor?

Ans. Height of Governor : It is the vertical distance from the centre of the ball to a point where the axis of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by 'h'.



Watt Governor

Height of Watt Governor : The simplest from of a centrifugal governor is the Watt Governor, as shown above. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways :

(i) The pivot, P may on the spindle axis as shown in 'a'.

(ii) The pivot, P may be offset from the spindle axis and the arms when produced intersect at O.

(iii) The pivot P, may be offset, but the arms crosses the axis at O.

Let M = Mass of the ball in kgs.

W = Weight of the ball in Newtons = mg

T = Tension in the arm in Newtons.

ω = Angular velocity of the arm and ball about the spindle axis in rad/s.

r = Radius of the path of rotation of the ball, i.e., horizontal distance from the centre of the ball to the spindle axes in metres.

F_c = Centrifugal force acting on the ball in Newtons

$= m\omega^2 r$ and

h = Height of governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the wt of the balls. Now the ball is in equilibrium under the action of,

(i) The centrifugal force (F_c) acting on the ball.

(ii) The tension (T) in the arm.

(iii) The weight (W) of the ball.

Taking moments about point O, we have

$$F_c \times h = W \times r = mgr$$

Or $$m\omega^2 rh = mgr$$

$\Rightarrow$ $$h = g/\omega^2$$

When g is expressed in m/s^2 and ω in the rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

$$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres}$$

Limitations of Watt Governor : At high speeds, the value of 'h' is small. At such speeds, the change in the value of 'h' corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds i.e., from the 60–80 r.p.m.

Q. 5. (b) Each arm of a Portergovernor is 400 mm long. The upper arms are pivoted on the axis of the sleeve and the lower arms are attached to the sleeve at a distance of 40 mm from the axis. Each ball has a mass of 6 kg and the weight on the sleeve is 50 kg.

Find the range of speed of the governor if the extreme radii of rotation of the balls are 260 mm and 300 mm.

Ans. BP = BD = 400 mm; DH = 40 mm; m = 6 kg; M = 50kg; $r_1 = 260$ mm and $r_2 = 300$ mm.

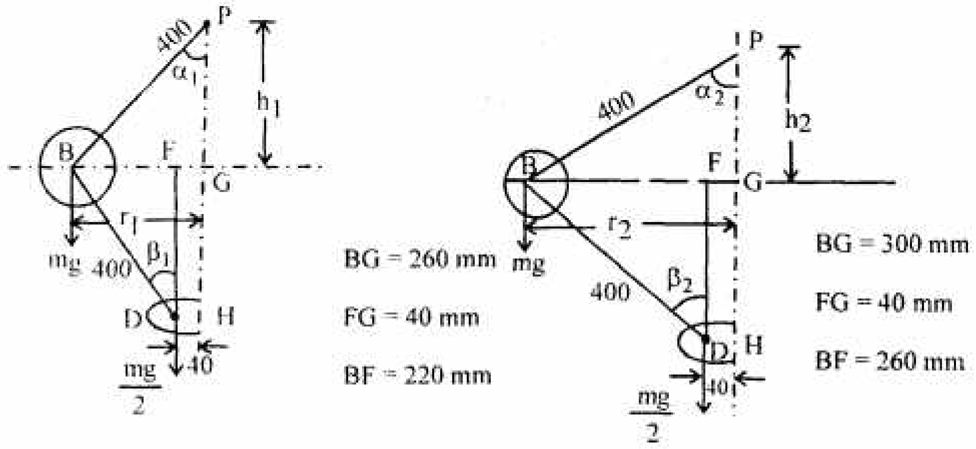
First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown by fig. (a) & (b) respectively.

Let N_1 = Minimum speed when r_1 = BG = 260 mm ; and

N_2 = Maximum speed when r_2 = BG = 300 mm.

From fig. (a), We find that the maximum height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2}$$



(a) (b)

$$. = \sqrt{(400)^2 - (260)^2} = \sqrt{92400} = 304 \text{ mm}$$

$$\boxed{h_1 = 0.3 m}$$

$$BF = BG - FG = 260 - 40 = 220 \text{ mm}$$

& $$DF = \sqrt{(DB)^2 - (BF)^2}$$

$$DF = \sqrt{(400)^2 - (220)^2} = \sqrt{111600} = 334 \text{ mm}$$

$\therefore$ $$\tan\alpha_1 = \frac{BG}{PG} = \frac{260}{304} = 0.855$$

$$\tan\beta_1 = \frac{BF}{DF} = \frac{220}{334} = 0.659$$

$\therefore$ $$q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.659}{0.855} = 0.771$$

We know that, $$(N_1)^2 = \frac{m + \frac{M}{2}(1+q_1)}{m} \times \frac{895}{h_1}$$

$$= \frac{6 + \frac{50}{2}(1+0.771)}{6} \times \frac{895}{0.3}$$

$$= \frac{6+25(1.771)}{6} \times \frac{895}{0.3}$$

$$N_1^2 = 2.50 \times 10^4$$

$$\boxed{N_1 = 1.58 \times 10^2 = 158\,\text{rpm}}$$

In fig. (b)

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{400^2 - 300^2}$$

$$\Rightarrow \quad h_2 = \sqrt{160000 - 90000} = \sqrt{70000}$$

$$h_2 = 2.65 \times 10^2\,\text{mm} = 0.265\,\text{m}$$

$$\boxed{h_2 = 0.265\,\text{m}}$$

$$BF = BG - FG = 260\,\text{mm}$$

$$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{400^2 - 260^2} = \sqrt{160000 - 67600}$$

$$DF = \sqrt{92400} = 3.04 \times 10^2\,\text{mm} = 0.304\,\text{m}$$

$$\Rightarrow \quad \boxed{DF = 0.304\,\text{m}}$$

$$\therefore \quad \tan\alpha_2 = BG / PG = \frac{300}{265} = 1.13$$

& $$\tan\beta_2 = \frac{BF}{DF} = \frac{260}{304} = 0.86$$

& $$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.86}{1.13} = 0.757 \Rightarrow \boxed{q_2 = 0.757}$$

We know that,

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q_2)}{m} \times \frac{895}{h_2}$$

$$= \frac{6 + \frac{50}{2}(1+0.757)}{6} \times \frac{895}{0.265}$$

$$N_2^2 = 28100$$

$$\boxed{N_2 = 168\,\text{rpm}}$$

We know that the range of speed

$$= N_2 - N_1 = 168 - 158 = 10\,\text{rpm}$$

$\therefore$ $\boxed{\Delta N = 10\,\text{rpm}}$ **Ans.**

Q. 6. (a) What is the difference between absorption and transmission dynamometers? What are torsion dynamometers?

Ans. Dynamometer : A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Differences between Absorption and Transmission Dynamometers :

Following are the two types of dynamometers, used for measuring the brake power of the engine :

(i) Absorption Dynamometer.

(ii) Transmission Dynamometers.

The main points of difference between these two are :

(a) In the absorption dynamometers, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement.

In the transmission dynamometers, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

(ii)

Absorption Dynamometers

Prony brake dynamometer — Rope brake dynamometer

Transmission dynamometer

Epicyclic train dynamometer — Belt transmission dynamometer — Torsion dynamometer

Torsion Dynamometer : A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. When the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (T), length of the shaft (l), diameter of the shaft (D) and modulus of rigidity (C) of the material of the shaft.

We know that the torsion equation is,

$$\frac{T}{J} = \frac{C\theta}{l}$$

Where, θ = Angle of twist in radians and

J= Polar moment of inertia of shaft.

For a solid shaft of diameter D, the polar moment of inertia,

$$J = \frac{\pi}{32} \times D^4$$

& for a hollow shaft of external diameter 'D' & internal diameter 'd', the polar moment of inertia,

$$J = \frac{\pi}{32}\left(D^4 - d^4\right)$$

From the above torsion equation,

$$T = \frac{CJ}{l} \times \theta = K\theta$$

Where, $K = \frac{CJ}{l}$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque & hence the power transmitted may be determined.

Q. 6. (b) A torsion dynamometer is fitted on a turbine shaft to measure the angle of twist. It is observed that the shaft twists 1.5° in a length of 5 metres at 500 r.p.m. The shaft is solid and has a diameter of 200 mm. If the modulus of rigidity for the shaft material is 85 GPa, find the power transmitted by the turbine.

Ans. Given,

$$\theta = 1.5° = \frac{1.5 \times \pi}{180} = 0.026\,\text{rad}$$

$$l = 5\,\text{m}$$

$$N = 500\,\text{r.p.m.}$$

Shaft is solid

$$D = 200\,\text{mm} = 0.2\,\text{m}$$

$$C = 85\,\text{GPa} = 85 \times 10^9\,\text{N/m}^2$$

We know that the polar moment of inertia for a solid shaft,

$$J = \frac{\pi}{32} \times D^4 \left(m^4\right)$$

$$J = \frac{3.14}{32} \times (0.2)^4 = 0.000157\,\text{m}^4$$

$$= 1.57 \times 10^{-4}\,\text{m}^4$$

& torque applied to the shaft,

$$T = \frac{C.J.}{l} \times \theta = \frac{85 \times 10^9 \times 1.57 \times 10^{-4}}{5} \times 0.026$$

$$\Rightarrow \quad T = 0.69394 \times 10^5\,\text{N}-\text{m} = 69.394 \times 10^3\,\text{N}-\text{m}$$

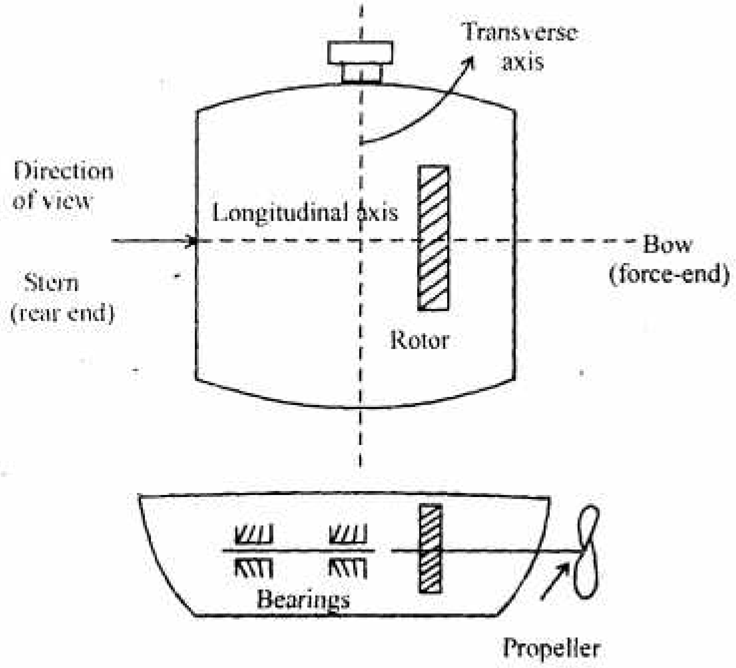
We know that the power of the engine,

$$P = \frac{T \times 2\pi N}{60} = \frac{69.39 \times 10^3 \times 2 \times 3.14 \times 500W}{60} = 3632 \times 10^3 W$$

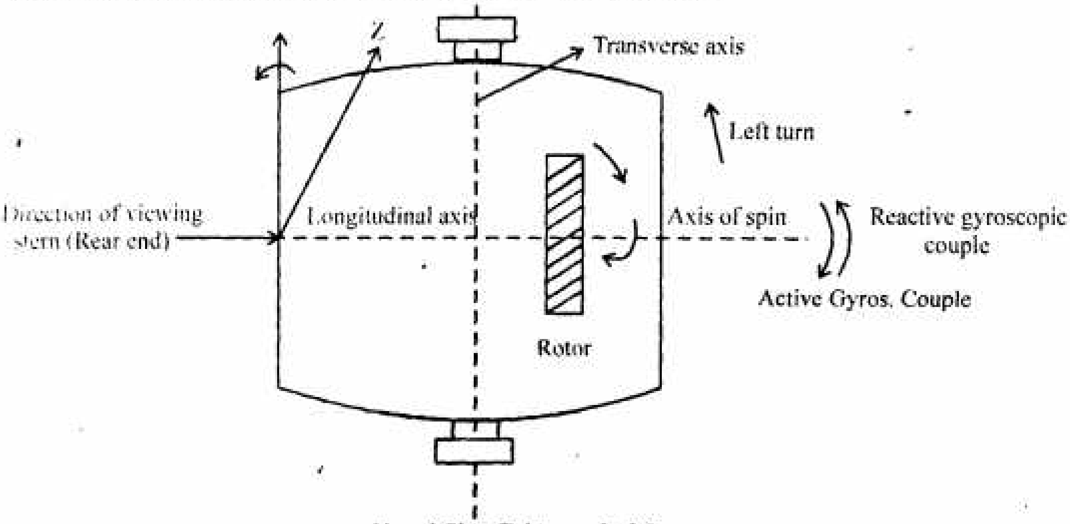
$\boxed{P = 3632\,kW}$ **Ans.**

Q. 7. (a) Discuss the gyroscopic effect on seagoing vehicles.

Ans. Gyroscopic Effect on Seagoing Vehicles (Ship) :



(i) Effect of Gyroscopic Couple on a Naval Ship During Steering :



Naval Ship Taking a Left Turn

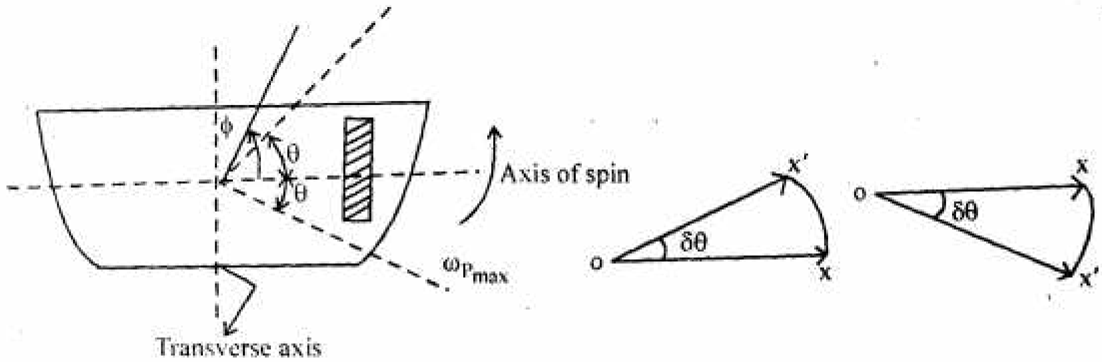
Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn and rotor rotates in the CW direction when viewed from stern.

When the rotor of the ship rotates in the CW direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown below in Fig. (a).

(a) *(b)*

As the ship steers to the left, the active G.C will change the angular momentum from ox to ox'. The vector xx' now represents the active gyroscopic couple and is perpendicular to ox. Thus, the plane of active G.C. is perpendicular to xx' and its direction in the axis oz for left hand turn is clockwise as shown. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (i.e., the anticlockwise direction). The effect of this reactive gyroscopic couple is to raise the bow and lower the stern.

(ii) Effect of Gyroscopic Couple on a Naval Ship During Pitching : Pitching is the movement of a complete ship up and



(a) *(b) Pitching upward* *(c) Pitching downward*

Effect of G.C. on a Naval Ship During Pitching :

down in a vertical plane about transverse axis as shown in fig. 'a'. In this case, the transverse axis is the axis of precision. The pitching of the ship is assumed to take place with S.H.M.

∴ Angular displacement of the axis of spin from mean position after time 't' seconds,

$$\theta = \phi \sin \omega_1 t$$

Where, ϕ = Amplitude of swing

ω_1 = Angular velocity of S.H.M. $= \dfrac{2\pi}{t_p}$ rad / s

t_p = Time period of S.H.M. in seconds.

Angular velocity of precision

$$\omega_p = \frac{d\theta}{dt} = \frac{d}{dt}(\phi \sin \omega_1 t)$$

$$\omega_p = \phi \omega_1 \cos \omega_1 . t$$

∴ Maximum angular velocity of precision.

$$(\omega_p)_{max} = \phi \omega_1 = \phi \times 2\pi / t_p$$

Let I = Moment of inertia of the rotor in kg-m^2 &

ω = Angular velocity of the rotor in rad/s.

∴ Maximum G.C.; $\quad C_{max} = I\omega(\omega_p)_{max}$

When the pitching is upward, the effect of the reactive G.C. (fig. b), will try to move the ship toward stare board. On the other hand, if the pitching is downward, the effect of the reactive G.C. in (fig. c) is to turn the ship towards port side.

(iii) Effect of Gyroscopic Couple on a Naval Ship During Rolling : For the effect of G.C. to occur, the axis of precision should always be perpendicular to the axis of spin. If, however, the axis of precision becomes parallel to the axis of spin, there will be no effect of the G.C. acting on the body of the ship.

In case of rolling of a ship, the axis of precision (i.e., longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the G.C. acting on the body of the ship.

Q. 7. (b) A 2.2 tonne racing car has a wheel base of 2.4 m and a track of 1.4 m from the rear axle. The equivalent mass of engine parts is 140 kg with radius of gyration of 150 mm. The back axle ratio is 5. The engine shaft and flywheel rotate clockwise when viewed from the front. Each wheel has a diameter of 0.8 m and a moment of inertia of $0.7\ kg - m^2$. Determine the load distribution on the wheels when the car is rounding a curve of 100 m radius at a speed of 72 km/hr to the left.

Ans. Given data,

M = 2200 kg

a = 1.4 m

l = 2.4 m

h = Height of C.G. of vehicle above ground,

[NOT GIVEN; QUESTION SOLVED WITH ASSUMPTION 'h']

l_1 = Distance of C.G. from front axle

l_2 = Distance of C.G. from rear axle.

[NOT GIVEN SO QUESTION IS SOLVED WITH THE ASSUMPTION OF C.G. BEING EQUIDISTANT FROM BOTH FRONT AND REAR AXLE]

$\therefore \quad l_1 = \frac{l}{2} = \frac{2.4}{2} = 1.2\text{ m}$

$\Rightarrow \quad \boxed{l_1 = 1.2\text{m}}$

Also $\quad \boxed{l_2 = 1.2\text{m}}$

$I_w = 0.7\text{ kg} - \text{m}^2$

$M_e = 140\text{ kg}$

$K_e = 150\text{ mm} = 0.15\text{ m}$

$\therefore \quad I_e = M_e K_e^2 = 140 \times (0.15)^2 = 3.15\text{ kg} - \text{m}^2$

$\Rightarrow \quad \boxed{I_e = 3.15\text{ kg} - \text{m}^2}$

$i = 5$

$R = 100\text{ m}$

V = Linear speed of vehicle $= 72\text{ km/h} = 72 \times \frac{5}{18}$

$\Rightarrow \quad \boxed{V = 20\text{ m/s}}$

The forces accounting for the stability of the vehicle are :

(i) Weight of the vehicle W = mg, giving rise to upward reaction at each wheel.

(ii) Precision of vehicle.

(a) Dead Weights : The weight of vehicle 'W' will be equally distributed over the four wheels which will act downwards.

The reaction between each wheel and the road surface of the same magnitude will act upwards. Therefore, Read reaction over each wheel

$$= \frac{W}{4} = \frac{mg}{4} \text{ Newtons}$$

$$\Rightarrow \quad \frac{W}{4} = \frac{2200 \times 9.8}{4} = 5390\text{N (upwards)}$$

(b) Centrifugal Couple :

$$C_C = \frac{MV^2h}{R} = \frac{2200(20)^2 h}{100} = 8800h\text{ N} - \text{m}$$

This couple is balanced by the vertical reactions at the four wheels being positive at the outer wheels and negative at the inner wheels.

$$\therefore \quad \frac{Q}{2} = \frac{C_C}{2a} = \frac{8800h}{2 \times 1.4} = 3140\,h\,N$$

This is vertically upwards for outer and downwards for inner wheels.

(c) Gyroscopic Couple :

(i) Due to wheels,

$$\omega_W = \frac{V}{r_\omega} = \frac{20}{0.4} = 50\,rad/s$$

$$\omega_{PW} = \frac{V}{R} = \frac{20}{100} = 0.2\,rad/s$$

$$\therefore \quad C_{gw} = 4I_w.\omega_W\omega_{PW}$$

$$C_{gw} = 4 \times 0.7 \times 50 \times 0.2 = 28N - m$$

$$\frac{P}{2} = \frac{C_{gW}}{2a} = \frac{28}{2 \times 1.4} = 10N$$

This reaction is vertically downwards on the inner wheels and upwards on the outer wheels.

(ii) G.C. due to engine parts,

$$\omega_e = i\omega_W\,rad/s = 5 \times 50 = 250\,rad/s$$

$$\omega_{pe} = \omega_{PW}$$

$$\therefore \quad C_{ge} = I_e\omega_e\,\omega_{Pe}$$

$$C_{ge} = 3.15 \times 250 \times 0.2 = 158N - m$$

$$\frac{F}{2} = \frac{C_{ge}}{2l} = \frac{158}{2 \times 2.4} = 32.9\,N \simeq 33N$$

This reaction is vertically downwards on the front wheels and upwards on the rear wheels. Now for left turn, the reactions are,

Front (outer 1) $= \frac{W}{4} + \frac{P}{2} + \frac{Q}{2} + \frac{F}{2}$

$= 5390 + 10 + 3140h + 33$

Front (outer 1) $= 5433 + 3140\,h$

Front (inner 2) $= \frac{W}{4} - \frac{P}{2} - \frac{Q}{2} - \frac{F}{2}$

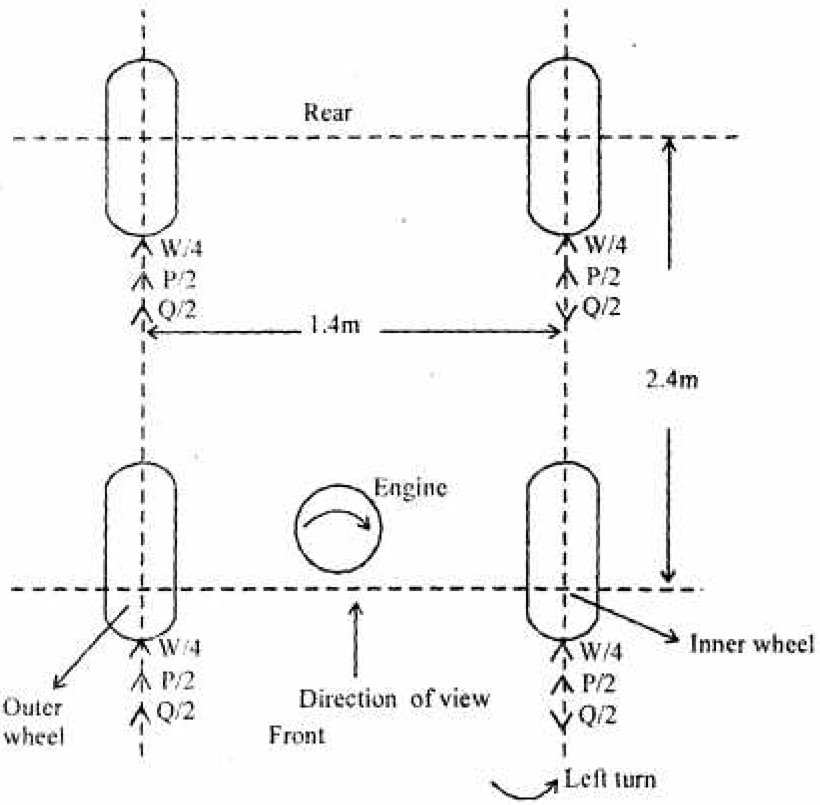
$= 5390 - 10 - 3140h - 33 = 5347 - 3140h$

Rear (outer) 3 $= \frac{W}{4} + \frac{P}{2} + \frac{Q}{2} + \frac{F}{2} = 5433 + 3140h$

Rear (inner) 4 $= \frac{W}{4} - \frac{P}{2} - \frac{Q}{2} - \frac{F}{2}$

Rear (inner) 4 $= 5347 - 3140h$

The load distribution for the above case is shown as below :



Q. 8. Write short notes on any three of the following :

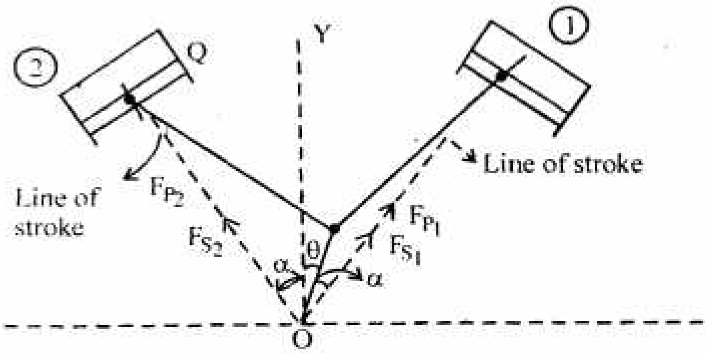
(a) Balancing of v-engines

(b) Stability of two wheel vehicles moving on curved paths

(c) Balancing Machines

(d) Engine shaking forces

Ans. (a) **Balancing of v-engines :** Consider a two cylinder v-engine as shown below :



Balancing of V-Engines

The common crank OC is driven by two connecting rods PC and QC. The line of stroke OP & OQ are inclined to the vertical OY, at an angle α .

Let $\quad$ m = Mass of reciprocating parts per cylinder,

l = Length of connecting rod,

r = Radius of crank,

n = Ratio of length of connecting rod to crank radius

$\Rightarrow \quad n = l / r$

θ = Inclination of crank to the vertical at any instant.

ω = Angular velocity of crank.

We know that inertia force due to reciprocating parts of cylinder 1, along the line of stroke,

$$= m\omega^2 r\left[\cos(\alpha - \theta) + \frac{\cos 2(\alpha - \theta)}{n}\right]$$

& the inertia force due to reciprocating parts of cylinder 2, along the line of stroke,

$$= m\omega^2 r\left[\cos(\alpha + \theta) + \frac{\cos 2(\alpha + \theta)}{n}\right]$$

The balancing of v-engines is only considered for primary and secondary forces as discussed below :

Considering Primary Forces : We know that primary force acting along the line of stroke of cylinder 1,

$$F_{P_1} = m\omega^2 r\cos(\alpha - \theta)$$

$\therefore$ Component of F_{P_1} along the vertical line OY,

$$= F_{P_1}\cos\alpha = m\omega^2 r\cos(\alpha - \theta)\cos\alpha \qquad ...(i)$$

& component of F_{P_1} along the horizontal line OX,

$$= F_{P_1}\sin\alpha = m\omega^2 r\cos(\alpha-\theta)\sin\alpha \qquad \text{...(ii)}$$

Similarly, primary force acting along the line of stroke of cylinder 2,

$$F_{P_2} = m\omega^2 r\cos(\alpha+\theta)$$

$\therefore$ Component of F_{P_2} along the vertical line OY,

$$= F_{P_2}\cos\alpha = m\omega^2 r\cos(\alpha+\theta)\cos\alpha \qquad \text{...(iii)}$$

& component of F_{P_2} along the horizontal line OX',

$$= F_{P_2}\sin\alpha = m\omega^2 r\cos(\alpha+\theta)\sin\alpha \qquad \text{...(iv)}$$

Total component of primary force along the vertical line OY,

$$F_{PV} = (i)+(ii) = m\omega^2 r\cos\alpha[\cos(\alpha-\theta)+\cos(\alpha+\theta)]$$

$$= m\omega^2 r\cos\alpha + 2\cos\alpha\cos\theta = 2m\omega^2 r\cos^2\alpha\cos\theta$$

and total component of primary force along the horizontal line OX,

$$F_{PH} = (ii)-(iv) = m\omega^2 r\sin\alpha[\cos(\alpha-\theta)-\cos(\alpha+\theta)]$$

$$= m\omega^2 r\sin\alpha + 2\sin\alpha\sin\theta$$

$$= 2m\omega^2 r\sin^2\alpha\sin\theta$$

$\therefore$ Resultant Primary force,

$$F_P = \sqrt{(F_{PV})^2+(F_{PH})^2}$$

$$\Rightarrow \qquad F_P = 2m\omega^2 r\sqrt{\left(\cos^2\alpha\cos\theta\right)^2+\left(\sin^2\alpha.\sin\theta\right)^2} \qquad \text{...(v)}$$

Considering Secondary Forces : We know that secondary forces acting the L.O.S. of cylinder 1,

$$F_{S_1} = m\omega^2 r\frac{\cos\alpha(\alpha-\theta)}{n}$$

$\therefore$ Component of F_{S_1} along the vertical line OY

$$= F_{S_1}\cos\alpha = m\omega^2 r\frac{\cos 2(\alpha-\theta)}{n}\times\cos\alpha \qquad \text{...(vi)}$$

& component of F_{S_1} along the horizontal line OX

$$= F_{S_1} \sin\alpha = m\omega^2 r \frac{\cos 2(\alpha - \theta)}{n} \times \sin\alpha \qquad \text{...(vii)}$$

Similarly, secondary force acting along the line of stroke of cylinder 2,

$$F_{S_2} = m\omega^2 r \frac{\cos 2(\alpha + \theta)}{n}$$

Component of F_{S_2} along the vertical line OY,

$$= F_{S_2} \cos\alpha = m\omega^2 r \frac{\cos 2(\alpha + \theta)}{n} \cos\alpha \qquad \text{...(viii)}$$

& component of F_{S_2} along the horizontal line OX'

$$= F_{S_2} \sin\alpha = m\omega^2 r \frac{\cos 2(\alpha + \theta)}{n} \times \sin\alpha \qquad \text{...(ix)}$$

Total component of secondary force along the vertical line OY,

$$F_{SV} = (\text{vii}) + (\text{viii}) = \frac{m}{n} \times \omega^2 r \cos\alpha\left[\cos 2(\alpha - \theta) + \cos 2(\alpha + \theta)\right]$$

$$F_{SV} = \frac{m}{n} \omega^2 r \cos\alpha \times 2\cos 2\alpha \cos 2\theta$$

$$F_{SV} = \frac{2m}{n} \omega^2 r \cos\alpha \cos 2\alpha \cos 2\theta$$

& total component of secondary force along horizontal line OX,

$$F_{SH} = (\text{vii}) - (\text{ix}) = \frac{m}{n} \omega^2 r \sin\alpha\left[\cos 2(\alpha - \theta) - \cos 2(\alpha + \theta)\right]$$

$$= \frac{m}{n} \omega^2 r \sin\alpha \times 2\sin 2\alpha \sin 2\theta$$

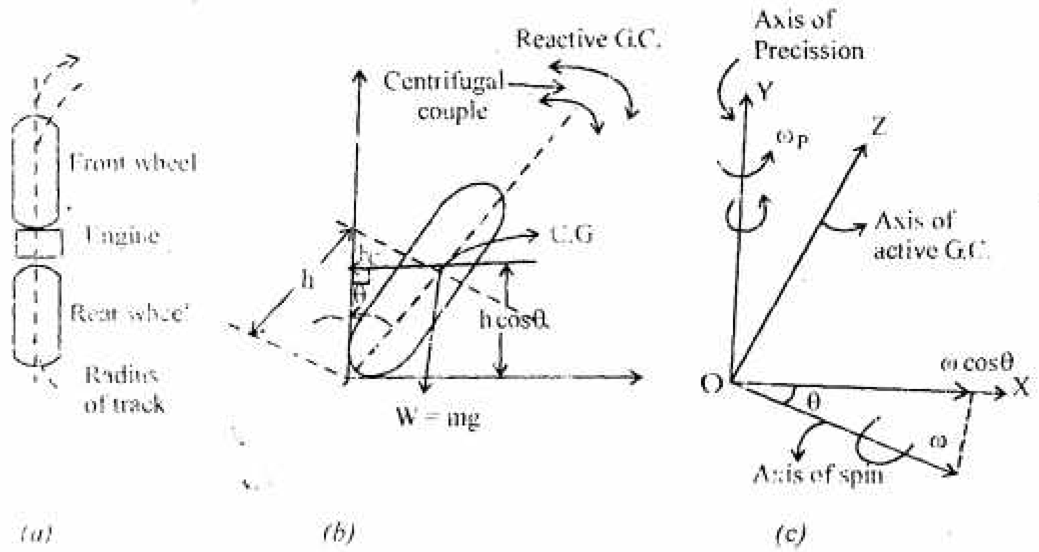
$$= \frac{2m}{n} \omega^2 r \sin\alpha \sin 2\alpha . \sin 2\theta$$

$\therefore$ Resultant secondary force $= F_S = \sqrt{(F_{SV})^2 + (F_{SH})^2}$

$$\Rightarrow \qquad F_S = \frac{2m}{n} \times \omega^2 r \sqrt{(\cos\alpha \cos 2\alpha \cos 2\theta)^2 + (\sin\alpha \sin 2\alpha \sin 2\theta)^2}$$

(b) Stability of Two Wheel Vehicles Moving on Curved Paths :

Consider a two wheel vehicle (say a scooter or motor cycle) taking a right turn on a curved path as shown below :



Stability of Two Wheel Vehicle Taking a Turn :

Let m = Mass of vehicle and its rider in kg.

W = Weight of the vehicle and its rider in newtons

$W = mg$

h = Height of C.G. of the vehicle and rider,

r_W = Radius of the wheels.

R = Radius of track or curvature.

I_W = Mass moment of inertia of each wheel.

I_E = Mass moment of inertia of the rotating parts of the engine.

ω_W = Angular velocity of the wheels.

ω_E = Angular velocity of the engine.

G = Gear ratio $= \omega_E / \omega_W$

V = Linear velocity of the vehicle $= \omega_W \times r_W$

θ = Angle of heel. It is the inclination of the vehicle to the vertical for equilibrium.

(i) Effect of Gyroscopic Couple : We know that,

$$V = \omega_W \times r_W$$

Or

$$\omega_W = V / r_W$$

& $$\omega_E = G.\omega_W = G\frac{V}{r_W}$$

$\therefore$ Total $$(I \times \omega) = 2I_W \times \omega_W \pm I_E \times \omega_E$$

$$= 2I_W \times \frac{V}{r_W} \pm I_E \times G \times \frac{V}{r_W} = \frac{V}{r_W}(2I_W \pm G.I_E)$$

& velocity of precision,

$$\omega_P = \frac{V}{R}$$

When the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in 'b'. This angle is K/n as angle of heel. In other words, the axis of spin is inclined to the horizontal at an angle θ, as shown in 'c'. Thus, the angular momentum vector $I\omega$ due to spin is represented by OA inclined to OX at an angle θ. But the precision is vertical. Therefore the spin vector is resolved along OX.

$\therefore$ Gyroscopic couple,

$$C_I = I\omega\cos\theta + \omega_P = \frac{V}{r_W}(2I_W \pm G.I_E)\cos\theta \times \frac{V}{R}$$

$$= \frac{V^2}{R.r_W}(2I_W \pm G.I_E)\cos\theta$$

(ii) Effect of Centrifugal Couple : We know that centrifugal force, $F_c = \frac{mV^2}{R}$.

This force acts horizontally through C.G. along the outward direction.

$\therefore$ Centrifugal couple, $$C_2 = F_C \times h\cos\theta = \left(\frac{mV^2}{R}\right)h\cos\theta$$

Since the centrifugal couple has a tendency to overturn the vehicle, therefore.

Total overturning couple,

$$C_O = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$C_O = \frac{V^2}{R.r_W}(2I_W + G.I_E)\cos\theta + \frac{mV^2}{R} \times h\cos\theta$$

$$= \frac{V^2}{R}\left[\frac{2I_W + G.I_E}{r_W} + mh\right]\cos\theta$$

We know that balancing couple $= mgh\sin\theta$

The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore, for stability, the overturning couple must be equal to the balancing couple. i.e.,

$$\frac{V^2}{R}\left(\frac{2I_W + GI_E}{r_W} + mh\right)\cos\theta = mgh\sin\theta$$

From this expression, the value of angle of heel (θ) may be determined, so that the vehicle does not skill.

(c) Balancing Machines : A balancing machine is a measuring tool used to balance rotating machine parts such as rotors for electronic motors, fans, turbines, disc brakes, disc drives, propellers and pumps. The machine usually consists of two rigid pedestals, with suspension and bearings on top supporting a mounting platform. The unit under test is bolted to the platform and is rotated either with a belt, air or end drive. As the part is rotated, the vibration in the suspension is detected with sensors and that information is used to determine the amount of unbalance in the part. Along with phase information, the machine can determine how much and where to add weights to balance the part.

There are two main types of balancing machines, hard bearing and soft bearing. The difference between them, however, is in the suspension and not the bearings.

In hard bearing machine, balancing is done at a frequency lower than the resonance frequency of the suspension. In a soft bearing machine, balancing is done at a frequency higher than the resonance frequency of the suspension.

A hard bearing machine is generally more versatile and can handle pieces with greatly varying weights, because hard bearing machines are measuring centrifugal forces and require only a one time calibration. Therefore, it works very well for low and middle size volume production and in repair workships.

A soft bearing machine is not so versatile with respect to amount of rotor weight to be balanced. It is suitable for high production volume and high precision balancing tasks.

Static balancing machines differ from hard and soft bearing machines in that the part is not rotated to take a measurement.

A blade balancing machine attempts to balance a part in assembly, so minimal correction is required later on.

Portable balancing machines are used to balance parts that cannot be taken apart and put on a balancing machine, usually parts that are currently in operation such as turbines, pumps and motors.

(d) Engine Shaking Forces : The reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce :

(i) Variation in tractive force along the line of stroke.

(ii) Swaying couple.

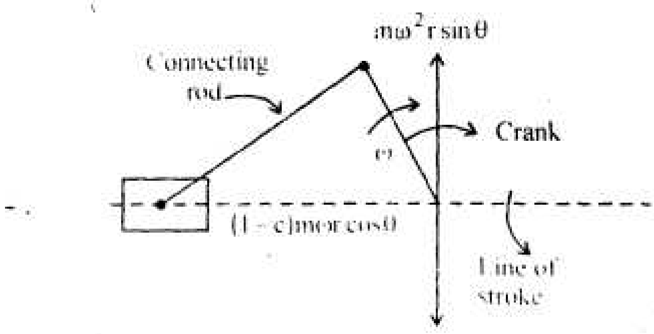
(iii) Hammer blow.

These are the three predominant engine shaking forces.

Variation of Tractive Force : The resultant unbalanced force due to the two cylinders along the line of stroke is known as tractive force. Let the crank angle for the first cylinder be inclined at an angle θ with L.O.S., as shown. Since the crank for the second cylinder is at right angle to the first crank, $\therefore$ the angle of inclination for the second crank will be $(90+\theta)$.

Let, M = Mass of reciprocating parts per cylinder

C = Fraction of the reciprocating parts to be balanced.



Variation of Tractive Force

We know that unbalanced force along L.O.S. for cycle 1,

$$= (1 - C)m\omega^2 r\cos\theta$$

Similarly, unbalanced force along the L.O.S. for cylinder 2,

$$= (1 - C)m\omega^2 r\cos(90 + \theta)$$

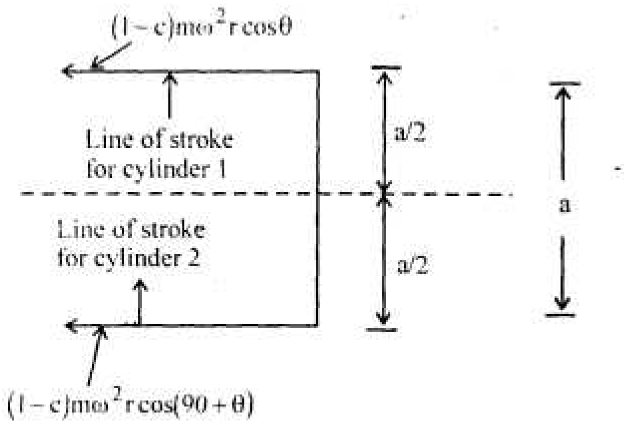
As per the definition,

F_T = Resultant unbalanced force along the L.O.S.,

$$F_T = (1 - C)m\omega^2 r\cos\theta + (1 - C)m\omega^2 r\cos(90 + \theta)$$

$$F_T = (1 - C)m\omega^2 r(\cos\theta - \sin\theta)$$

Swaying Couple : The unbalanced forces along the L.O.S. for the 2 cylinders constitute a couple about the centre line YY between the cylinders as shown :



Swaging Couple

This couple has swaying effect about a vertical axis and tends to sway the engine alternately in CW and ACW directions. Hence, the couple is known as the swaying couple.

Let a = Distance between the centre lines of the 2 cylinder.

$\therefore$ Swaying couple $= (1-C)m\omega^2 r\cos\theta \times \frac{a}{2} - (1-C)m\omega^2 r\cos(90+\theta)\frac{a}{2}$

$$= (1-C)m\omega^2 r \times \frac{a}{2}(\cos\theta + \sin\theta)$$

Hammer Blow : The maximum magnitude of the unbalanced force along the perpendicular to the L.O.S. is known as hammer below.

Hammer blow $= B\omega^2 b$

The effect of hammer blow is to cause the variation in pressure between the wheel and rail. The limiting condition in order that the wheel does not left from the rails is given by:

$$P = B\omega^2 b$$

& permissible value of the angular speed,

$$\omega = \sqrt{\frac{P}{B.b}}$$